The University of West Hungary
Kitaibel Pál Environmental Doctoral School
Geoenvironmental Sciences Program

Ph.D. thesis

SYNTHETIC MODELLING OF THE GRAVITATIONAL FIELD

JUDIT BENEDEK

Supervisor
Dr. GÁBOR PAPP

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Precedents and objectives of the research

The research, the results of which are presented in this thesis is the continuation of F014284[1] (finished in 1997) and T025318[2] (finished in 2001) OTKA programs. The first version of the 3D density model of the lithosphere of the Pannonian basin (Papp 1996a) was created in the framework of F014284 OTKA research program. That model used rectangular prism as volume element. Different versions of the geoid were computed by its application for the territory of Hungary. Such a 3D model makes possible – with specific conditions – to determine different parameters of the gravity field (gravitational acceleration, geoid undulation, gravitational potential, gravity anomaly) analytically. Furthermore, because of the rigorous functional relations between the parameters of the field created from the density model by forward modelling, it is possible to test numerical methods (for example a specific solution of the Stokes integral) which transform from one system of parameters to another one. In the T025318 OTKA research program the lithosphere model was extended to describe the Carpathian-Pannonian Region. One of the aims of this program was to determine the gravitational field lines going through the topographic masses using forward modelling techniques. Based on the model calculations it was possible to study the order of difference between the horizontal coordinates of a specific surface point obtained from GPS measurements (Helmert projection) and its projection point corresponding to its elevation coordinate determined by levelling (Pizetti projection). Furthermore, it became possible to compute the free-air gradient (second order partial derivatives in vertical direction of the gravity potential) using analytical methods and it was also possible to determine territorial distribution of deviations from its normal value within the Pannonian Basin (Csapó and Papp 2000). Based on the model used for the investigations detailed above I tested the effect of the point density of gravity data on the accuracy of geoid undulation determined by the Stokes-FFT method (Benedek 2000, 2001). For the test all geodetic boundary values (gravity anomalies) and also the boundary surface to be determined (geoid undulation) were derived analytically from the model. The OTKA program included the comparison of simulated deflections of the vertical data with the 138 available astro-geodetic deflections of the vertical data.

The OTKA studies mentioned above indicated that the refinement of the model is beneficial in some applications. One possibility is to apply a more realistic volume

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[1] OTKA Nr.: F014284 High precision modelling of the gravity field and geoid computation in Carpathian-Pannonian Region, 1994-1997, Principal Investigator: Papp Gábor

[2] OTKA Nr.: T025318 The effect of local characteristics of the Earth's gravity field on geodetic coordinates. Simulation studies in the Pannonian basin, 1998-2001, Principal Investigator: Papp Gábor

element which improves the geometrical description of the structural surfaces. Such a simple geometric element is the polyhedron, because it allows creating bodies bounded with oblique surfaces. Its application can provide a more realistic geometrical description of boundary surfaces, without height jump which is an unavoidable artifact of the description made by rectangular prisms. In this way, the artificial gravitational effect due to the step-like structure can be eliminated. The stepped structure mainly influences the second and the third order derivatives of the potential in near surface points. Furthermore, the effect of the Earth's curvature can be taken into consideration during computations, because the polyhedral geometry allows the description of this model in a global coordinate system (e.g. WGS84). Compared to the rectangular prisms the analytical formulas of the polyhedron’s gravitational potential and its higher order derivatives are more complicated so their calculation is more time consuming. Recently there is a HP A500 dual processor computer with a 64 bit architecture that provides the needed computational capacity at GGRI Hung. Acad. Sci., Sopron.

One aim of my thesis was to compare the contributions to geoid undulation and gravity anomaly synthetically computed from polyhedron and rectangular prism models describing the crustal structure of the Carpathian-Pannonian region. Furthermore, I wished to compare the second order vertical derivatives determined from these two models in near-surface points by forward modelling. In this comparison I wanted to involve in situ measurements (Csapó and Papp 2000) so I chose the Sóskút testing area of Tech. Univ. Budapest (TUB) as a target area for model computations. The third aim was to give an estimate of the gravitational contribution of the different geological units such as topography, upper mantle and Neogen-Quaternary sedimentary complex to the second derivatives of disturbing potential i.e. to the elements of the Eötvös tensor by using the extended, regional scale (Alpine–Pannonian–Carpathian region) lithosphere model. I wanted to study whether the on board gradiometric observations of the GOCE (Gravity and Steady-State Ocean Circulation Experiment) satellite could be applicable to deduce some regional information about the horizontal density variation of the crust as well. I aimed also to derive formulas for the transformation of the computed second derivatives of the potential in local (using the rectangular prism as volume element) and in global (using the polyhedron as volume element) coordinate systems. Based on this I also wanted to determine the effect of the Earth's curvature on field parameters.

I wished to study the analytical behaviour of the formula given for the gravitational potential of the polyhedron volume element and the numerical characteristics of its higher order derivatives in points close to and far form the effective source, and also the accuracy of model computation and the time-consumption of the analytical formulas.

Research methods and results

I applied the theorems of the potential theory to define the domains of formulas describing first and second derivatives of the gravitational potential and the potential itself generated by the polyhedrons.

Using vector analysis I derived analytical formulas for the gravitational potential and its first and second derivatives of the polyhedrons.

I developed a program system in HP Fortran language for computing the gravitational potential of the polyhedron and the first and second derivatives of the potential.

For the forward modelling of the gravity field related quantities I used different models of the topography of the Alpine–Pannonian–Carpathian (ALPACA) region built up with triangular prisms and polyhedrons as volume elements.

Computational and modelling results were compared to measurements in certain cases and I managed to show the advantages of using the polyhedron models.

I used different coordinate systems (planar and global) which are transformable into one-another. I applied various models (elementary and optimised) of rectangular prisms generated by different methods representing the local (planar) mapping system and polyhedron elements defined in the global Cartesian coordinate system. These models are also transformable into one-another (there is a one-to-one geometrical correspondence between the corner points of prisms and polyhedrons). Using these models I was able to estimate the effect of the Earth's curvature.

In my thesis I summarised and completed the analytical formulas of the gravitational potential and its first and second order derivatives of the polyhedron volume element. Using vector analysis I gave a uniform derivation for the formulas. I demonstrated that passing from surface integrals to contour integrals using either Gauss-Ostrogradsky or Stokes theorem leads to finding the same vector function, and to define it one have to solve a quasi-linear differential equation. I proved that the domains of the analytical formulas could be extended to the domains deducted from the potential theory, thus the singularities of the formulas can be eliminated.

I studied the numerical stability of the polyhedron-based model in points close to and far the effective source giving each point location those limits where 1) the analytical formulas become senseless or 2) the numerical error is dominating in computed value. As far as first derivative of the potential is concerned I completed the relation given by Holstein and Ketteridge (1996) and Holstein et al. (1999) with new relations about the potential and its second order derivatives. I showed by double precision computations that using the polyhedral model of the ALPACA region the error of the forward computation of the second order derivatives of the potential is less than 1% within the studied area.

I compared the runtimes of the potential and its derivatives applying the polyhedron algorithm and the cod for the rectangular prism optimised by D. Nagy (1988).

Based on a 5 km × 5 km DTM of the Carpathian Basin and a 500 m × 500 m DTM of Hungary I created polyhedron and different type (elementary and optimised) rectangular prism models for both territories. Both in the elementary rectangular prism and the polyhedron models horizontal dimensions of the volume elements is equal to the resolution of the relevant DTM, while the vertical dimensions of the volume elements are equal to the values given by the DTM point. Another rectangular prism model (optimised) was created based on the minimum number of volume elements principle (Kalmár et al. 1995). Using various models developed from a 5 km × 5 km DTM, the order of the average and the standard deviation of the computed gravitational disturbance at the geoid level is about -0.1 mGal and ± 0.5 mGal throughout the 800 km × 600 km computational area which includes also the territory of Hungary. In the second application I used models derived from 500 m × 500 m DTM, and the computation was made on a 165 km × 150 km territory of the Northern Mid Mountains Range. The computations which were performed at the geoid level show that the differences between the results computed from optimised prism and polyhedron models appear more remarkably on the low plains than on higher territories.

A big advantage of the polyhedron volume element is that it can describe the surface without jumps in height, so the second order derivatives of potential in z are much smoother and more realistic in near surface parts. It can be deduced from the study carried out on the Sóskút test area of TUB that in accordance with the theory the second order derivatives computed from polyhedron model at 1 m height above the topographic surface are correlating fairly well with topography. To model the second order derivatives of the potential in near-surface points the polyhedron volume element is needed. Even if the rectangular prism model is derived from a 10 m × 10 m DTM the variation of the derivatives between adjacent points (for example points of a 25 m × 25 m grid) can be too (unreal) high. Therefore the correlation between the values itself and the surface is low. In the six point of Sóskút geodetic network dedicated for studying deformation, vertical gradient (VG) values computed from the polyhedron model of the area fits well with the measurements (Csapó és Papp 2000) apart from a shift, but the values obtained from rectangular prism model are in contradiction with the measured values.

From results of the synthetic gravitational modelling for the planed orbit altitude (~250 km) of GOCE satellite it was found that the individual contribution of the topography and the upper mantle to the second derivatives of the disturbing potential reach 1 Eötvös. In case of the Neogene-Quaternary sediments this contribution is several hundredths of Eötvös unit only, but this is still higher than the projected measurement sensitivity. As the topography and the density distribution of the sediments are known much better than the density contrast at Moho–the boundary between the lower crust and the upper mantle – we can use their synthetically modeled contributions as correction in relation with the measurements at orbit altitude. Residuals (i.e GOCE measurements – corrections) can be converted into density values with inversion, so the density contrast at Moho surface can be estimated more precisely.

For forward computation of the contribution of the topography the polyhedron model is recommended (i.e. using global coordinate system) because of the effect of the Earth's curvature for this component is greater than the sensitivity of the satellite gradiometer. When computing the contribution of the effect of the sediments it is enough to use local coordinate system, i.e. rectangular prisms because the effect of the curvature is estimated to be in the order of the noise range. If 10% accuracy is enough, then the local system is sufficient for the inversion.

Summary of new scientific results

1. I defined the general solution $\mathbf{f}_i(\mathbf{r}_P) = \dfrac{\phi^*\left(\dfrac{y'}{x'}\right) + r_{MP}}{R_{MP}^2}\mathbf{R}_{MP}$ of that quasi-linear partial differential equation $\nabla_{\mathbf{r}_P}\mathbf{f}_i(\mathbf{r}_P) = \dfrac{1}{r_{MP}}$ which analytical formula of gravitational potential of polyhedron can be reduced to. In the general solution by choosing suitable function ϕ^* we can get the individual solutions of other authors back. I completed the existing analytical formulas for the first order derivatives of the potential given by other authors with formulas for the potential and the second order derivatives of the potential in case when these were not determined by them.

   I determined numerical stability ranges for constants involved in the analytical formulas of gravity potential and its first and second order derivatives. I also determined limiting values of constants in critical points and I set up a classification for the formulas determining the constants based on the needed computation time.

2. I gave an estimation and defined functions for the numerical errors of analytical formulas of potential and its derivatives. The errors are for the polyhedron function of normalised distance (normalisation was effectuated by linear dimension of polihedron) of the computation point. I handled the exponent appearing in these functions as a parameter. It was 2.2 and 3.0 in case of the potential and the second order derivatives of the potential applying the conditions of double precision computation and 100% error level. I showed that the numerical error is less than 1% either for far (at e.g. GOCE orbital altitude) or near surface (<1 m) points if the polyhedron model of the crustal structure of ALPACA (Alpine–Pannonian–Carpathian) region is used for forward computations.

3. I found a correlation between the time of the computation and the computational parameters (number of volume elements and computational points) of the polyhedron and rectangular prism model. The time needed for calculating gravity potential and its first order derivative with the algorithm developed by me is 1.5 times more using polyhedrons than the one optimised by D Nagy (1988) for the rectangular prisms, applying double precision arithmetic.

4. Modelling the vertical gradient of gravity (VG) in near-surface points based on DTM having 10 m $\times$ 10 m resolution the rectangular prism approach does not provide enough accuracy. The change of the second order derivatives by z of disturbing potential can be too high even between adjacent points (25 m). In near surface points the second order derivative by z reacts sensitively to the stepped structure of the modelled surface due to the geometry of the rectangular prism. Therefore the observed correlation between the surface and the gradients, the existence of which follows from the theory is very weak. The application of polyhedrons, however, may improve significantly the correlation even if the resolution of the basic DTM is not increased as it is proven by the computations on the Sóskút test are of TUB. I also showed that using polyhedron model, the changes of the computed VG (vertical gradient) values between the 6 network points correlate well with the changes of the VG values derived from in situ gravity observations.

5. By the application of forward modelling, I demonstrated that the individual contributions of the topography and of the upper mantle to the second derivatives of the disturbing potential T certainly reaches one Eötvös unit in the planned altitude (~250 km) of the GOCE (Gravity and Steady-State Ocean Circulation Experiment) satellite. The contribution is only several hundredths Eötvös in case of the Neogene-Quaternary sedimentary complex. Additionally, I found that in the

ALPACA region the effect of the Earth's curvature is an average of 10% of the absolute value of local contributions i.e. several hundredth Eötvös unit in the studied altitude range (300 km – 400 km). Considering the topography the effect of the Earth's curvature on the second order derivatives of the potential highly exceeds the sensitivity of the satellite gradiometer. In case of the sediments this effect is estimated to be within the expected noise range of the measurements.

I found also that when one eliminates the effect of topography and of the sediments from the measurements of the GOCE, the gradient observations can be transformed into density contrast value by means of inversion of the residual effect. It gives a real chance to increase the precision of the density contrast value at the Moho surface.

Utilizing the outcomes of the thesis

The result of these studies contradicts the general opinion that using polyhedrons in forward modelling is less efficient than the application of the mostly used rectangular prisms. Its reasoning is based on the complexity of formulas and the time consuming character of computations. By the presented studies we can affirm that usage of the two volume elements (the rectangular prism and the polyhedron) in computations complements one-another, because depending on the actual task both may have either advantage or drawback. The needed runtime of computing of the gravitational potential of the polyhedron and its higher order derivatives is one and half fold of the runtime of rectangular prism computations, but it can be reduced further by using optimal formulas. Depending on the actual object it has to be decided that between the conflicting criteria of accuracy and rapidity which one is to be favoured. As I demonstrated, in case of computing first or second order derivatives near to the source body the difference between the results computed with the two different volume elements can be significant. Therefore, if these parameters are to be simulated for e.g. comparison with observations only the application of polyhedrons may lead to acceptable results. I proved by numerical studies of that the formulas derived for computation of the gravitational effect of the polyhedron are numerically stable both in the vicinity of the source body and far from it (for example at the altitude of GOCE). The numerical error either in local or in regional scale is not greater than 1%. In case of the potential and its first and second order derivatives of polyhedron it is possible to estimate numerical error based on the functions derived by me.

In general we can say that in the vicinity of the dominant source body or near to the surface of a density jump, description of the gravity field parameters (geoid undulation, gravity disturbance, gradients of gravity) can be more accurate when the description of the boundary surface in the vicinity of the point is more detailed. One effective method for this can be the usage of the polyhedron.

The algorithm for the computation of the gravity potential and its higher order derivatives can be exploited in education for demonstrating gravity field of geological bodies, natural and artificial deposits and caves as well.

For modelling the density inhomogenities in the Earth (for example structural inhomogenities of lithosphere, natural deposits) and modelling the consequences in the surface and the subsurface space of the exogenous (mostly erosional) processes the polyhedron approach is more justified. In such cases the usage of the polyhedron for modelling together or instead of the rectangular prism should be considered.

Personal publications related to the Ph.D. work

Publications in refered journals: **15,** *Publications in SCI's journal:* **3,** *IF*: **1.81**
SCI reference: **20,** *Reference from other source*: **29.**

Papp G, Benedek J (1998): Numerical modelling of gravitational field lines. (in Hungarian) Publications in Geomatics, I: 55 - 70.

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Benedek J, Papp G (2006): Simulation of the Eötvös tensor elements at GOCE satellite altitude (in Hungarian) Publications in Geomatics, X.: 187-200.

Papp G, Szeghy E, Benedek J (2008): The determination of potential difference by the joint application of measured and synthetical gravity data: a case study in Hungary. Journal of Geodesy, DOI: 10.1007/s00190-008-0257-2.

Benedek J, Papp G (2009): Geophysical inversion of on board satellite gradiometer data: A feasibility study in the ALPACA Region, Central Europe. Acta. Geod. Geoph. Hung. (in pres).

Personal presentations related to the Ph.D work

1997. September 3 – 9, IAG Scientific Assembly, Rio de Janeiro, G Papp, J Benedek: Plumblines, normal and horizontal coordinates. A case study in the Pannonian Basin, Hungary, poster presentation.

1998. December 11. Winter Seminar on Engineering Geodesy, Sopron, J Benedek, G Papp: Numerical determination of gravitational field lines - the effect of mass attraction on horizontal coordinates, presentation.

1998. April 23 – 24, Conference of Young Scientists, Kecskemét, J Benedek, G Papp, J Kalmár: Numerical modelling of gravitational field lines (in Hungarian), presentation.

1998. May, Romanian Geological Institute, Bucharest, J Benedek, G Papp: Numerical determination of field lines in gravitational space, presentation.

2000. May 4 – 5, 8th International Meeting on “Alpine Gravimetry”, Leoben, J Benedek: The Effect of the Point Density of Gravity Data on the Accuracy of Geoid Undulations investigated by 3D Forward Modeling, presentation.

2000. July 31 - August 4, Geoid and Geodynamics, Calgary, Canada, J Benedek: The Effect of the Point Density of Gravity Data on the Accuracy of Geoid Undulations investigated by 3D Forward Modeling. Gravity, , poster presentation.

2000. October 11 – 12, Seminar on Geomatics, Sopron, J Benedek.: The Effect of the Point Density of Gravity Data on the Accuracy of Geoid Undulations computed by Stokes FFT method, presentation.

2001. September 2 – 7, IAG Scientific Assembly, Budapest, J Benedek: The Effect of the Point Density of Gravity Data on the Accuracy of Geoid Undulations investigated by 3D Forward Modelling, poster presentation.

2001. October 23 – 27, AROPA Workshop, Luxemburg, J Benedek: The gravitational potential and its derivatives for the prism: Theory and application, presentation.

2003. May 8 – 9, 1st Workshop on International Gravity Field Research, Graz, G Papp, D Nagy, J Benedek: On the information equivalence of gravity field related quantities – a comparison of gravity anomalies and deflection of vertical data., presentation.

2003. May 8 – 9, 1st Workshop on International Gravity Field Research, Graz, J Benedek: The application of polyhedron volume element in the calculation of gravity related quantities., presentation.

2004. October 28 – 29, Seminar on Geomatics, Sopron, J Benedek, G Papp: Evaluation of gravity measurements for instrument investigation (in Hungarian), presentation.

2004. October 28 – 29, Seminar on Geomatics, Sopron: G Papp., J Benedek: Application of gravity inversion for the investigation of the geoid (in Hungarian), presentation.

2005. April 24 – 29, European Geosciences Union (EGU), Wien, Austria, G Papp, J Benedek: Application of gravity inversion for the investigation of geoid definitions, poster presentation.

2006. Aug 28th – Sept 1st, The first Symposium of IGFS in Istanbul, J Benedek, G Papp: Geophysical inversion of on board satellite gradiometer data: A feasibility study in the ALPACA Region, Central Europe, poster presentation.

2006. 28th – Sept 1st, The first Symposium of IGFS in Istanbul, G Papp, J Benedek: Inverse and forward gravitational modelling for the investigation of the primary indirect/quasi-indirect effect of Helmert’s second method of condensation, presentation.

2006. October 26 – 27, Seminar on Geomatics, Sopron, J Benedek, G. Papp: Simulation of the Eötvös tensor elements at GOCE satellite altitude (in Hungarian), presentation.

2007. December 20, Session of the Scientific Commission of Geodesy of the Hung. Acad. Sci., Budapest: G Papp, J Benedek: Gravimetric research in Geodetic and Geophysical Research Institute of HAS, presentation.

2008. October 31, Memorial Day of István Széchenyi, Sopron: G Papp, J Benedek: The Eötvös tenzor and the satellite gradiometer, presentation.

2008. November 6 – 7, Seminar on Geomatics, Sopron: G Papp, J Benedek: Forward modelling using 3D models of the interior of the quarry of Ferőrákos and its topographic surroundings (in Hungarian), poster presentation.

2008. November 6 – 7, Seminar on Geomatics, Sopron: G Papp, J Benedek: High accuracy determination of geopotential number by the joint application of measured and synthetical gravity data, presentation.

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